

## Joint Source Channel Coding for Image Transmission over Wireless Channel Using Sequence Minimum Mean Squared Error Decoder

<sup>1</sup>Jalel Chebil and <sup>2</sup>Boualem Boashash

<sup>1</sup>Department of ECE, Faculty of Engineering, Hashemite University  
Zarqa 13115, Jordan

<sup>2</sup>SPR Labratary School of EESE, Queensland University of Technology  
Brisbane QLD 4001, Australia

**Abstract:** Transmission of image and video over time-varying wireless channel can benefit from the use of the joint source-channel (JSC) coding methods. This study investigates the use of the sequence-based approximate MMSE method in the improvement of the joint source channel decoding of a DPCM system. Using this technique, three methods are presented. The produced results show that the method with the best performance outperforms the sequence maximum a posteriori technique.

**Key words:** Joint source channel coding, sequence minimum mean-square error

### INTRODUCTION

In Today's multimedia systems which mix data, text, speech, audio, image and video, the different sources are compressed as much as possible before the bits are transmitted via the communication channels. Generally the higher the compression factor the higher is the sensitivity to channel errors. A single error in the received data might render the remainder of the bitstream useless. Furthermore, the decoding of the-now erroneously interpreted-bitstream will add to the distortion of the decoded image. Since in a mobile environment, the channel is quite noisy where an average error rates up to 10% are quite common<sup>[1]</sup>, both source and channel coding are used. According to Shannon's separation principle, these components can be designed independently without loss in performance<sup>[2,3]</sup>. However, this important theory is based on several assumptions that may break down in practice. Better performance can be obtained if joint source channel coding techniques are employed.

Joint source channel coding has become an important research topic and several approaches were proposed. These methods include optimised rate allocation, unequal error protection, optimised index assignment, channel optimised quantisation and recently exploiting the source residual redundancy<sup>[3-7]</sup>. The work presented in this manuscript falls into the category of joint source channel coders which use the residual redundancy in the output coder for improved reconstruction over noisy channel.

The term "residual redundancy" was used by Sayood and Borkenhagen<sup>[4]</sup> to refer to statistical dependency that remains in the output of a DPCM prediction loop. In<sup>[8]</sup>, residual redundancy was defined in the information-theoretic sense of excess rate. Methods based on residual redundancy do not attempt to remove excess rate from the source via entropy coding or improved prediction; rather, they use this redundancy as a form of implicit channel coding, to perhaps remove the need for explicit channel coding. A properly chosen JSC decoder can capitalize upon the excess rate. Two fundamental decoding approaches have been suggested for this purpose. One approach is Maximum A Posteriori (MAP) decoding and the second is Minimum Mean Squared Error (MMSE) decoding.

The MAP approach estimates the transmitted sequence output by the encoder and then feeds the estimated indexes to a standard decoder. However, the MAP structure is inherently suboptimal. Examples of MAP approaches: sequence MAP detection<sup>[8]</sup> and<sup>[5]</sup>, Instantaneous MAP detection<sup>[8]</sup> and Modified MAP receiver<sup>[9]</sup>.

For the MMSE approach, the JSC decoder directly acts as conditional mean estimator, the MMSE estimator for the source as shown in Fig. 1. Examples of such approaches: Instantaneous approximate minimum mean squared error (IAMMSE) decoder, sequence-based approximate MMSE with first order and second order Markovian approximation (SAMMSE)<sup>[10]</sup>.

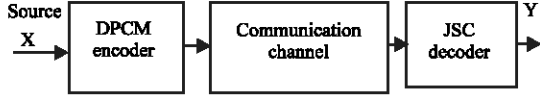


Fig. 1: System model

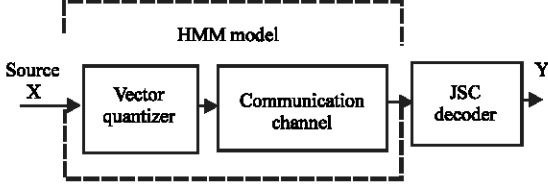


Fig. 2: HMM models used in the SAMMSE decoder, I: sequence of hidden states, J: observed discrete symbol

In this study, our objective is to design a source decoder which minimizes the mean squared error in the reconstruction of a linear autoregressive DPCM coded signal over a noisy channel as shown in Fig. 1. The proposed sequence MMSE is based on the sequence-based approximate MMSE with first order Markovian approximation algorithm.

**Sequence-based approximate MMSE:** The SAMMSE method combines the advantage of the sequence based aspect of MAP decoding and the conditional mean aspect of the instantaneous MMSE decoding. Given the memoryless channel and Markovian index sequence assumptions, the source encoder and channel tandem is effectively considered to be discrete hidden Markov model. In this model, the possible transmitted indexes correspond to the hidden states and the received corrupted indexes correspond to the observed symbols produced within these states. Fig. 2 shows a block diagram of the model, where, a vector quantizer is used as source encoder. A detailed description of this method is given below.

Consider the sequence  $\underline{Y} = (y_1, y_2, \dots, y_T)$  which the decoder uses to approximate the source sequence  $\underline{X} = (x_1, x_2, \dots, x_T)$ . The SAMMSE method determines the values of  $\underline{Y}$  that minimize the expected distortion given a sequence of received indexes,  $\underline{J}$ .

The distortion incurred by  $\underline{Y}$  is given by

$$D(\underline{X}, \underline{Y}) = \sum_{t=1}^T \|x_t - y_t\|^2 \quad (1)$$

The expected distortion can be expressed as

$$E[D(\underline{X}, \underline{Y}) \setminus \underline{J} = \underline{j}] = \sum_{\underline{I}} [D(\underline{X}, \underline{Y}) \setminus \underline{I} = \underline{i}] = \frac{\Pr\{\underline{I} = \underline{i}, \underline{J} = \underline{j}\}}{\Pr\{\underline{J} = \underline{j}\}}$$

$$\frac{\sum_{\underline{I}} \left( \sum_{t=1}^T E[d(x_t, y_t) \setminus \underline{I} = \underline{i}] \right) \Pr\{\underline{J} = \underline{j} \setminus \underline{I} = \underline{i}\} \cdot \Pr\{\underline{I} = \underline{i}\}}{\sum_{\underline{I}} \Pr\{\underline{J} = \underline{j} \setminus \underline{I} = \underline{i}\} \cdot \Pr\{\underline{I} = \underline{i}\}} \quad (2)$$

The quantities  $\Pr\{\underline{J} = \underline{j} / \underline{I} = \underline{i}\}$  and  $\Pr\{\underline{I} = \underline{i}\}$  can be computed using the assumption that the communication channel is memoryless and the transmitted sequence is first-order Markovian. They can be written as

$$\Pr\{J=j \setminus I=i\} = \prod_{t=1}^T P\{J_t = j_t \setminus I_t = i_t\}$$

and

$$\Pr\{I=i\} = \Pr\{I_1 = i_1\} \cdot \prod_{t=2}^T \Pr\{I_t = i_t \setminus I_{t-1} = i_{t-1}\}$$

However, choosing values of  $y_t$  by directly minimizing Eq. 2 still involves  $E[d(x_t, y_t) \setminus \underline{I} = \underline{i}]$  which requires a decoder table of cardinality  $N^T$  for each  $t$ . A direct computation from (2) will involve a huge number of multiplications; hence, this approach is not practical. To reduce the amount of computation, Miller *et al.*<sup>[8]</sup> suggested a first order approximation

$$E[d(x_t, y_t) \setminus \underline{I} = \underline{i}] \approx E[d(x_t, y_t) \setminus I_t = i_t]$$

where  $t$  can take any value between 1 and  $T$ . For this study, minimizing Eq. 2 is equivalent to selecting  $y_t$  for each  $t$  to minimize the quantity

$$\hat{D} = \sum_{t=1}^T \sum_{l=1}^N \|y(l) - y_t\|^2 \cdot \Pr\{I_t = l \setminus \underline{J} = \underline{j}\} \quad (3)$$

where

$$y(l) = E[X_t \setminus I_t = l], \quad (4)$$

and

$$\Pr\{I_t = l \setminus J = j\} = \frac{\sum_{\underline{I}} \Pr\{\underline{J} = \underline{j} \setminus \underline{I} = \underline{i}\} \cdot \Pr\{I_t = l\}}{\sum_{\underline{I}} \Pr\{\underline{J} = \underline{j} \setminus \underline{I} = \underline{i}\} \cdot \Pr\{I_t = l\}} \quad (5)$$

The optimal decoder in the sense of  $\hat{D}$  selects the reconstructions according to the centroid rule

$$y_t = \sum_{l=1}^N y(l) \cdot \Pr\{I_t = l \mid \underline{J} = \underline{j}\}, \text{ for all } t \quad (6)$$

The key to realizing Eq. 6 is to find an efficient way of calculating the probabilities in (5). The discrete HMM method provides such method. In this study, the source encoder and channel tandem is interpreted as a discrete hidden Markov model, with the unknown sequence  $\underline{I}$  as sequence of hidden states and with the received index  $\underline{j}$ , the observable discrete symbol produced in a hidden state at time  $t$ . Therefore  $\Pr\{I_t = l \mid \underline{J} = \underline{j}\}$  can be computed by the well-known forward/backward algorithm. If we define forward probabilities  $\alpha_t(l) = \Pr\{j_1, j_2, \dots, j_t, I_t = l \mid \lambda\}$  and backward probabilities  $\beta_t(l) = \Pr\{j_{t+1}, j_{t+2}, \dots, j_T \mid I_t = l, \lambda\}$  based on the discrete HMM parameters  $\lambda = (A, B, \Pi)$ , where  $A$  is the state transition probability matrix,  $B$  is the probability of the observed symbol in a given state and  $\Pi$  is the initial state distribution. These probabilities can be calculated through the standard forward/backward recursions which are

$$\alpha_t(l) = \Pr\{I_t = l\} \Pr\{J_t = j_t \mid I_t = l\}, \quad l=1, 2, \dots, N$$

$$\alpha_t(l) = \left( \sum_{k=1}^N \alpha_{t-1}(k) \cdot \Pr\{I_t = l \mid I_{t-1} = k\} \right) \cdot \Pr\{J_t = j_t \mid I_t = l\},$$

for  $l=1, 2, \dots, N$  and  $t=2, 3, \dots, T$

$$\beta_T(l) = 1, \quad \text{for } l=1, 2, \dots, N$$

$$\beta_t(l) = \sum_{k=1}^N \Pr\{I_{t+1} = k \mid I_t = l\} \cdot \Pr\{J_{t+1} = j_{t+1} \mid I_{t+1} = k\} \cdot \beta_{t+1}(k)$$

for  $l=1, 2, \dots, N$  and  $t=T-1, \dots, 1$ .

Finally, the a posteriori probabilities are computed via

$$\Pr\{I_t = l \mid \underline{J} = \underline{j}\} = \frac{\alpha_t(l) \cdot \beta_t(l)}{\sum_{m=1}^N \alpha_t(m) \cdot \beta_t(m)} \quad (7)$$

In summary, the SAMMSE algorithm can be described by the following major steps

- Compute the HMM parameters  $A$  and  $\Pi$  from the training set of images
- Determine the HMM parameter  $B$  from the channel characteristics
- for  $l=1, 2, \dots, N$  and  $t=1, 2, \dots, T$ 
  - Evaluate  $\alpha_t(l)$  and  $\beta_t(l)$
  - Compute  $\Pr\{I_t = l \mid \underline{J} = \underline{j}\}$  from (7).
- For  $l=1, 2, \dots, N$ , compute  $y(l) = E[X_t \mid I_t = l]$  from the codebook
- Determine  $y_t$  from (6) for all  $t$

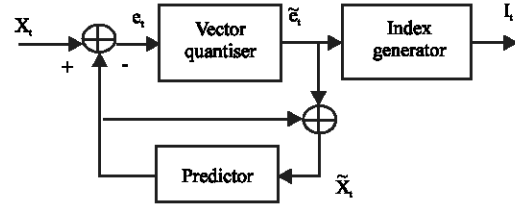


Fig. 3: DPCM encoder with auto regressive prediction

**Application of the SAMMSE to DPCM system:** In this study, we apply the SAMMSE decoding technique in the reconstruction of DPCM signals over noisy channel. We focus on the DPCM systems with auto regressive prediction. This is due to the popularity of these systems and the fact that the ideas employed in this study can be easily applied to other study including moving average predictive (linear or nonlinear) encoding systems. Fig. 3 shows the block diagram of a DPCM encoder with first order auto regressive prediction. For a source input  $X_t$ , the quantized sample,  $\hat{X}_t$ , is given by

$$\hat{X}_t = \hat{e}_t + A_1 \cdot \hat{X}_{t-1} \quad (8)$$

where  $\hat{e}_t$  represents the quantized value of the predictive error  $e_t$ .

The SAMMSE algorithm as described in previous study was developed for the study when a vector quantizer is used as a source coder. In order to apply the algorithm to a DPCM system, some modifications have to be introduced. The output sequence,  $I_t$ , produced by the DPCM encoder is actually the output of the vector quantization of the predictive error  $e_t$ . However, in the SAMMSE model,  $I_t$  is the output of the vector quantization of  $X_t$ . Considering this difference, three approaches are proposed.

**First approach:** In the first approach, the same notation is used as in the previous study. Therefore, the MMSE decoder outputs,  $y_t$ , will approximate the prediction error  $e_t$  and  $X_t$  can be estimated from  $y_t$  using Eq. 8

$$\begin{aligned} \hat{X}_t &= \hat{e}_t + A_1 \cdot \hat{X}_{t-1} \\ &= y_t + A_1 \cdot \hat{X}_{t-1} \end{aligned} \quad (9)$$

Hence, for the first approach, the modified algorithm can be written as following

- Compute the HMM parameters  $A$  and  $\Pi$  from the training set of images
- Determine the HMM parameter  $B$  from the channel characteristics

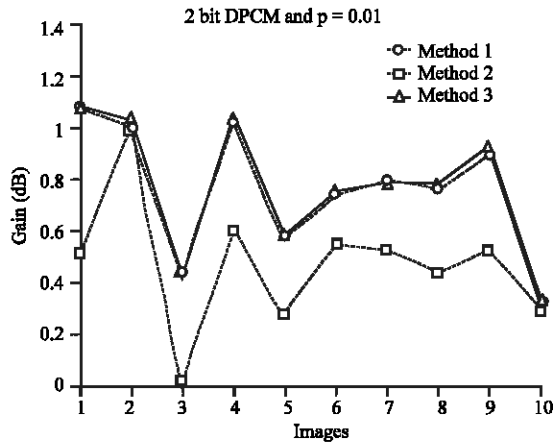


Fig. 4: Improvement obtained by the three methods for the 2 bit DPCM and p = 0.01

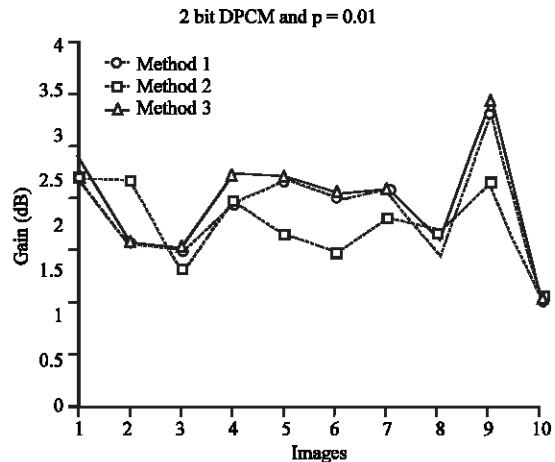


Fig. 5: Improvement obtained by the three methods for the 2 bit DPCM and p = 0.1

- for  $l=1,2,\dots, N$  and  $t=1,2,\dots, T$ 
  - Evaluate  $\alpha_l(l)$  and  $\beta_l(l)$
  - Compute  $\Pr \{I_t=1 \setminus J=j\}$  from (7).
- For  $l=1,2,\dots, N$ , compute  $y(l)$  from the codebook
- Determine  $y_t = \hat{e}_t$  from (6) for all  $t$ .
- Compute  $\hat{X}_t$  from (9)

**Second approach:** The second method is similar to the first one except in the last step, equation (13),  $y_t$  is replaced by its closest value to the codebook.

**Third approach:** In the third method,  $y_t$  is considered to represent the estimated value of  $X_t$ . In such study,  $y(l)$  is interpreted differently from the previous methods and it is expressed as  $y(l) = E[X_l \setminus I_t=1] = A_1 \cdot \hat{X}_{t-1} + \text{codebook}(l)$ . Therefore, the algorithm becomes

- Compute the HMM parameters  $A$  and  $\Pi$  from the training set of images
- Determine the HMM parameter  $B$  from the channel characteristics
- for  $l=1,2,\dots, N$  and  $t=1,2,\dots, T$ 
  - Evaluate  $\alpha_l(l)$  and  $\beta_l(l)$
  - Compute  $\Pr \{I_t=1 \setminus J=j\}$  from (7).
- For  $l=1,2,\dots, N$ , compute  $y(l)$
- Compute  $y_t = \hat{X}_t$  from (6) for all  $t$ .

## RESULTS AND DISCUSSION

The system described in Fig. 1 was implemented for various study using MATLAB software. In the first part of the experiment, the proposed methods were tested and compared. For this purpose, twelve standard images are used such as couple, lena, etc. The images are of size  $256 \times 256$  or  $512 \times 512$  and with 8 bits per pixel. Three images are used for training and the rest are employed for testing. Each image was encoded to 2-bit and 3-bit DPCM samples and was sent over the channel. The communication channel was assumed to be a binary symmetric channel with bit error probability  $p$  ranging from 0.01 to 0.20.

The initial state probabilities,  $\Pi$  and state transition probabilities,  $A$ , were estimated from the encoded sequence of the three training images. The performance of each proposed method was evaluated using the Reconstruction Signal-to-Noise Ratio (RSNR) measure which is defined as<sup>[5]</sup>.

$$RSNR = 10 \log_{10} \frac{\sum 255^2}{\sum (x_i - \hat{x}_i)^2} \quad (10)$$

where  $x_i$  is the input to the source coder and  $\hat{x}_i$  is the output of the source decoder.

Figure 4 to 6 show the improved obtained by the three proposed methods for the SAMMSE technique for

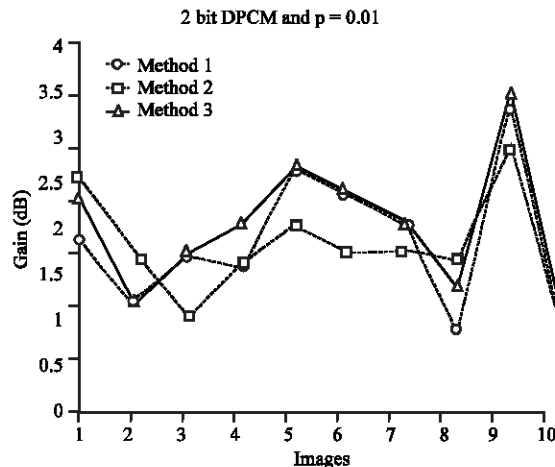


Fig. 6: Improvement obtained by the three methods for the 2 bit DPCM and p = 0.15

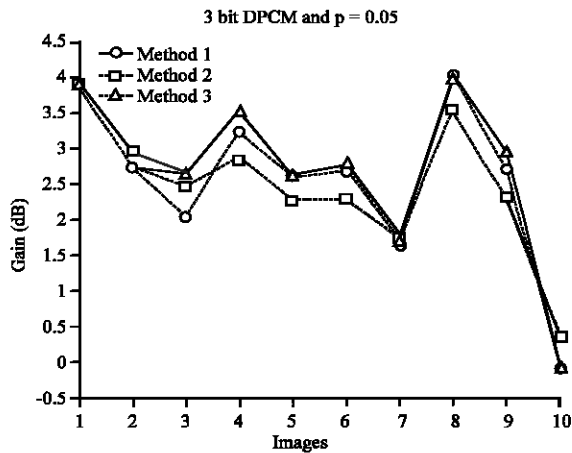


Fig. 7: Improvement obtained by the three methods for the 3 bit DPCM and  $p = 0.05$

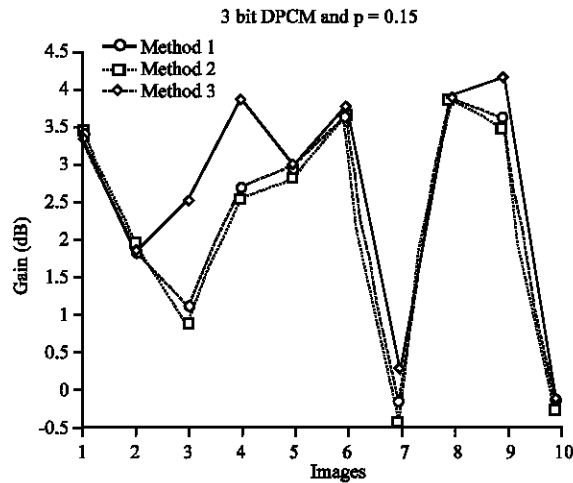


Fig. 8: Improvement obtained by the three methods for the 3 bit DPCM and  $p = 0.15$

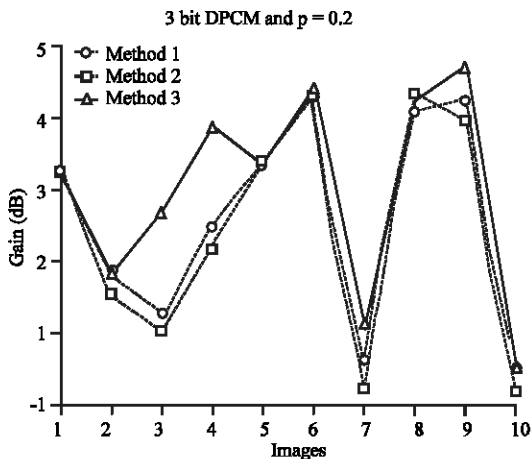


Fig. 9: Improvement obtained by the three methods for the 3 bit DPCM and  $p = 0.20$

the study of 2 bit and for various values of  $p$ . The results indicate that in general the third method produce better solution. For the study of 3 bit DPCM, we observed similar results a shown in Fig. 7-9.

When the third method is compared with the sequence MAP technique. It was found that it outperforms the sequence MAP technique. For example, using image “Cameraman” for 2 bit DPCM system and  $p=0.10$ , 0.77 dB improvement was obtained with sequence MAP in comparison of 2.75 dB obtained with the third method. For the study of image “Clown” and with the same condition, the proposed method outperformed the sequence MAP by 1.5 dB.

### CONCLUSION

This study studies the use of the sequence-based approximate MMSE method in the improvement of the joint source channel decoding of a DPCM system. Based on this technique, three methods were proposed and tested. The method with the best performance is selected and compared with the sequence MAP technique. It was found that the best method outperform sequence MAP technique.

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