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Closed-Form Optimal Batch Size Equation to Minimize Process Lead Time for Single Machine

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Abstract: Batch sizing strategy in the manufacturing system has significant impacts on the production performance. Most researchers used some complicated techniques such as simulation, queuing theory, optimization model or complex system equations to find the optimal batch size. Applying those techniques to assess the effects of the change in the production factors such as demands and capacity on the process performance requires some customizations and is difficult. In this study, we proposed closed-form optimal batch size equations that can be easily used to assess the impact of changes in production volume. The number of batches from the optimal batch size equation may not be integer but they provide the lower bound on the process lead time.

Key words: Closed-form, optimal batch size, minimize lead time, single machine, Thailand

INTRODUCTION

Batch sizing strategy in the manufacturing system has significant impacts on the production performance. The process with large batches usually has long lead time, low inventory turnover, large amount of finished goods inventory and more safety stocks are required in order to maintain the on-time delivery performance. Moreover, more working capitals and assets will be tied to those finished goods inventories. Nevertheless, when the demands increase and production capacity remains the same, producing in large batches is inevitable.

Numbers of studies offered the approaches to find or estimate the optimal batch size that improve the process efficiency such as lead time, service quality and costs. Parija and Sarker (1999) developed the cost functions and mixed integer programming to find the optimal batch size that minimized the total costs of raw materials ordering and finished goods inventory. However, the time intervals or cycle time interval between shipments are fixed. Wang and Chen (2005) modified the cost function of Parija and Sarker to find the possible solutions and used simulation model to estimate the optimal batch size in the supply chains system. Bertrand (1985) extended the Economic Order Quantity (EOQ) and queuing model to find the optimal batch size in "made-to-stock" environment. However, the algorithm is an iterative procedure that the batch size will be determined by adjusting the parameters such as production rates, wait time, idle cost, in process inventory cost and work orders for each iteration. Wang and Sarker (2004) used mixed integer nonlinear programming to determine the optimal batch size in the kanban system. Ojha*et al*. (2007) developed the model to optimize batch size for an imperfect production system with quality assurance and rework. A total cost equation was developed for the model and the optimal ordering quantities were evaluated. Rau and Yang (2008) introduced an optimal batch size for integrated production-inventory policy in a supply chain. The objective was to minimize the joint total costs incurred by the vendor and the buyer. First, the researchers developed a mathematical model and proved that it had the optimal solution. Then, they describe an explicit solution procedure for obtaining the optimal solution.

In this study, we present simple version of batch sizing model for a single process system that is easy to use. The objective is to derive a closed-form batch size equation that minimizes the overall process lead time.

MATERIALS AND METHODS

Assumptions and notation: To develop the proposed equation, the following assumptions and notation are used.

Assumptions: Demands are deterministic and constant over the evaluation period. However, if the demands are not constant, we can divide the evaluation period into several smaller periods. Each period has constant demands but not necessary be the same rate among

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different intervals. Then we can independently apply the optimal batch size equation for each period. For simplicity, we assume that the demands are constant throughout the evaluation period. Setup times are deterministic and independent, i.e., the set up time does not depend on what has been made before. There is no maximum or minimum limit on batch size. There are no integrality and non-negative restrictions.

Notation:
Sets
A = Set of all items

Coefficients and parameters:
D_i = Total demand for item i
D = Total demand (D $\sum_{i\in A} D_i$)
p_i = Processing time per unit of item i
s_i = Setup time per batch of item i
s = Available setup time
T = Available machine hours in the evaluation period

Decision variables:
B_i = Batch size of item i
n_i = Total number of batches of item i required to produce D_i units. Note that $n_i = \frac{D_i}{B_i}$ t and $B_i = \frac{D_i}{n_i}$

Batch size equation that minimizes overall process lead time: In this study, we estimate overall process lead time by weighted average cycle time interval. If B_i is feasible batch size of item i, the Cycle Time Interval (CTI) of item i is Eq. 1:

$$CTI_i = \frac{B_i}{D_i} \quad (1)$$

We use the demands for the weights for the weighted average cycle time interval which is defined in Eq. 2:

$$\text{Weighted average CTI} = \frac{\sum_{i\in A} CTI_i \cdot D_i}{\sum_{i\in A} D_i} \text{ or } = \frac{1}{D}\cdot\sum_{i\in A} CTI_i \cdot D_i \quad (2)$$

The objective is to find the feasible batch sizes of all products that minimize the overall process lead time. Regardless the cost information, the decision is based on the demands, processing time, setup time and available capacity. The results from this equation may not provide the cheapest total cost; in this case holding costs and setup costs but it might be useful when we want to establish the base line for many analyses, e.g., the trade-off between lowest lead time and cheapest costs. We can rewrite the weighted average CTI in Eq. 2 as the function of number of batches as shown in Eq. 3:

$$\text{Weighted average } CTI_i = \frac{1}{D}\cdot\sum_{i\in A} CTI_i \cdot D_i = \frac{1}{D}\cdot\sum_{i\in A} B_i = \frac{1}{D}\cdot\sum_{i\in A}\frac{D_i}{n_i} \quad (3)$$

With the assumptions that the demands or sales quantities are given, the total processing time is predeterministic; therefore, we can simplify the capacity constraint to that the total setup time cannot exceed available setup time s = T. Total processing time:

$$T - \sum_{i\in A} D_i \times p_i$$

In addition, if the available setup time is not sufficient to produce at least one batch of each item, i.e.,:

$$s < \sum_{i\in A} s_i$$

the problem may not be feasible. The number of batches would be fraction less than one or negative number. The objective function and constraint can be defined as followed:

$$\text{Min}\frac{1}{D}\times\sum_{i\in A}\frac{D_i}{n_i} \quad (4)$$

$$\sum_{i\in A} n_i \times s_i \le s \quad (5)$$

Since, there is only one constraint, we solve this problem by using Lagrangian multiplier technique. The Lagrangian equation (L) is defined below as:

$$L = \frac{1}{D}\cdot\sum_{i\in A}\frac{D_i}{n_i} + \lambda\cdot\left(\sum_{i\in A} n_i \cdot s_i - s\right) \quad (6)$$

Minimizing Eq. 6 and solving for n_i and λ that satisfy the First Order Conditions (FOC) give:

$$\lambda = \frac{1}{\delta^2 \cdot D}\cdot\left(\sum_{i\in A}\sqrt{D_i \cdot s_i}\right)^2 \quad (7)$$

$$n_i = \frac{s_i}{\sum_{i\in A}\sqrt{D_j \cdot s_i}}\cdot\sqrt{\frac{D_i}{s_i}} \quad (8)$$

$$B_i = \frac{D_i}{n_i} = \left(\frac{\sum_{i\in A}\sqrt{D_i \cdot s_i}}{s}\right)\cdot\sqrt{s_i \cdot D_i} \quad (9)$$

The derivation and FOC are derived in. Although, the numbers of batches obtained from the minimum batch size equation are usually not integer, we can use them to establish the lower bound of the process lead time. They also imply the production frequency of the item in a production cycle. For example, if we have 10 batches of item A, 20 batches of item B and 40 batches of item C, the frequency ratio is 1/10:1/20:1/40: = 0.1:0.05: 0.025 or 4:2:1. That is item C will be made every cycle, item B will be made every two cycles and item A will be made every four cycles.

Another interesting result from solving Eq. 6 is λ which is the dual variable associated with the capacity constraint in the original problem. It can be interpreted as a "shadow price" of available setup time that can be used to quickly estimate the impact of overall processing time reduction that is equivalent to obtaining additional available setup time to overall process lead time. The value of λ is corresponded to the planning period and unit of available setup time. For example, if the planning horizon is 1 year and unit of available setup time is hours, the λ implies the reduction in overall CTI in years given a small increment in available setup time in hours. In addition, λ is always positive and capacity constraint Eq. 5 is always binding constraint since the lead time is always shorter in the smaller batch production.

RESULTS AND DISCUSSION

Assume that there are five items in the production plan. Their demands and production parameters are given in Table 1. From Table 1, total processing time required to produce all products is 5,037.55 h. If the available machine hours in a year are 7,500.00 h and total processing hours (without setup time) are 5,037.55 h, the available setup time is 2,462.45 h. Using the minimum lead time batch size equation yields 15.09 days of overall process lead time. The number of batches and batch sizes are shown in Table 2.

Assume that there were 360 day in 1 year. The λ value for this instance is 0.000017. This implies that if total processing time has been reduced by 1 h that is equivalent to obtaining one additional available setup time, the process lead time will be decreased by 0.000017 year or 0.0061 day or 8.784 min.

Application of the batch size equation: In this study, we will illustrate how to apply the optimal batch size equations to quickly assess some strategic issues such as the impact of the change in total production volume. When the total production volume is increased, one way to obtain more productivity without adding more capacity (or more available machine hours) is to increase the batch sizes. It is equivalent to decreasing the number of batches and that reduces the total setup time required. In other words, we can spend some of the setup time saved on production. Producing in large batches may have adverse impacts on finished goods inventory and processing lead time, since each batch will take longer time to complete.

Conversely, if the total production volume is decreased, total processing time required will be decreased or we will have more available setup time. Consequently, each product can be produced in smaller batches that lower the finished goods inventory and process lead time. We use the example for a base case where the % change in demand is 0%. Using simple "what-if" analysis tool, "one-way data table" in Microsoft Excel, Fig. 1 was created. Figure 1 shows the change in CTI and finished goods inventory when the total production volume changed. From Fig. 1, we used weighted average CTI (Weighted average CTI = or weighted by demand) to measure the overall process CTI. The average finished goods inventory for each product is

Table 1: Total demand and production parameters

	Item				
Variables	1	2	3	4	5
Processing time/unit (p)	0.25	1.25	1.8	0.5	2
Setup time/batch (s)	20	30	15	25	20
Annual Demand (D)	258	1105	1126	1130	500

Table 2: Minimum lead time batch size

	Item				
Variables	1	2	3	4	5
Total batches (n)	13.57	22.92	32.73	25.39	18.89
Batch size (B)	19.02	48.2	34.41	44.5	26.48
CTI (days)	26.54	15.7	11	14.18	19.06

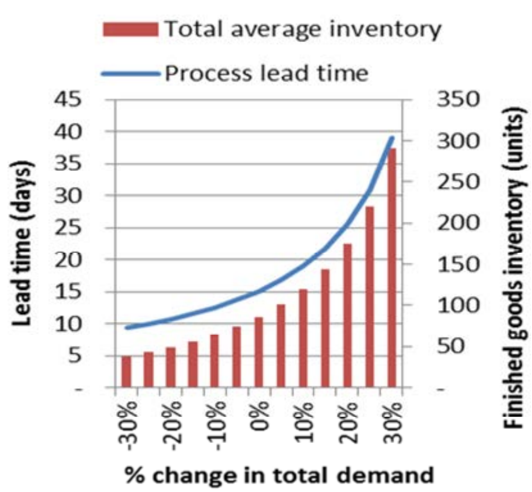

Fig. 1: The impact of the change in total production volume to lead time and finished goods inventory

half of its batch size, since the demand is constant. The % change in total production will be applied across all items, i.e., Demand for part i = (1+% Change)×Original demand for item i. The weighted average CTI and total average finished goods inventory increase when the total production volume increases. The changes become even more sensitive when the production is reaching the maximum capacity. If the information on additional sales revenue is available, we can use this trade-off curve to perform cost-benefit analysis whether the additional sales revenues from adding more production volumes worth the loss in flexibility or the increase in lead time and more assets will be tied to the increase in finished goods inventory.

CONCLUSION

In this study, we presented closed-form optimal batch size equation. The optimal batch size equation is the generalized version of the typical process that they can be used to estimate the process lead time associated with the size of the batch given the demand. We also used the optimal batch size equation to explain the impact of increasing production volume.

APPENDIX 1

Solving Lagrangian equation for minimum overall process lead time batch size equation:

$$L = \frac{1}{D} \cdot \sum_{i \in A} \frac{D_i}{n_i} + \lambda \cdot \left(\sum_{i \in A} n_i \cdot s_i - s \right)$$

First order conditions:

$$\frac{\partial L}{\partial \eta_i} = -\frac{1}{D} \cdot \frac{D_i}{\eta_i^2} + \lambda \cdot s_i = 0; \text{ for all items}$$

$$\frac{\partial L}{\partial \lambda} = \sum_{i \in A} n_i \cdot s_i - s = 0$$

Rearrange the first equation of FOC:

$$n_i^2 = \frac{D_i}{\lambda \cdot s_i \cdot D}$$

Since, n≥0, substitute $n_i = \sqrt{D_i / \lambda \cdot s_i \cdot D}$ the second equation of FOC and solve for λ:

$$\lambda = \frac{1}{s^2 \cdot D} \cdot \left(\sum_{i \in A} \sqrt{D_i \cdot s_i} \right)^2$$

From the equation, the optimal λ is always positive if at least one s_i is positive. This implies that the capacity constraint in the original problem always the binding constraint or:

$$\sum_{i \in A} n_i \cdot s_i = s$$

Substitute ë in the first equation of FOC and solve for n_i:

$$n_i = \frac{s}{\sum_{i \in A} \sqrt{D_i \cdot s_i}} \cdot \sqrt{\frac{D_i}{s_i}}$$

And minimum batch size for item i is $B_i = D_i/n_i$.

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