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The Interaction of Oncoming Compression Shocks

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Abstract: In this research, we review the interaction of oncoming compression shocks. The criteria of transition from regular reflection of oncoming shocks to irregular: Von Neumann criterion and criterion of stationary mach configuration are described. The regions in which the transition from one type of reflection to another is only possible by a shock and regions of fluent transition are described. Dependencies of reflected shock intensity on intensity of interacting oncoming shocks are presented. A range of the problem parameters where the existence of regular and irregular reflection of shock is assumed, explains the possibility of hysteresis.

Key words: Compression shock, increase, interaction, mach reflection, shock wave, structure, transformation

INTRODUCTION

Aim of the research is to present main data on the problem of interaction between oncoming compression shocks. The easiest way to present oncoming gas-dynamic discontinuities is to present them in a one-dimensional case, when shock waves move towards each other (Fig. 1a). Depending on their intensity, the various type of wave interaction can occur (Uskov, 2000). In analogy to one-dimension case, the interacting shock that turn flow into different directions (Fig. 1b) are called oncoming oblique compression shocks. The need to study oncoming shock arises during design of modern air intakes of internal compression that required operating at high mach number, ramjets with subsonic and supersonic combustion asymmetrical supersonic nozzles and other cases.

In some sense, this problem is a generalization of oblique shock reflection from a wall or symmetry plane case (Viktorovich and Nikolaevich, 2014). The difference is in that the flow picture can be asymmetrical, i.e., the interacting shocks (shock waves) in general case, case have different intensities. As a result, the pictures of oncoming shock interaction differ in great variability than in the case of wave reflection from a wall. Molder (1960) have developed an analytical theory of regular interaction of oncoming shock waves. As in the case of shock reflection from a wall the regular and irregular interaction of oncoming compression shocks are possible (Uskov, 1980). The greatest investment into determination of existence regions of various regular and irregular interaction of oncoming shocks belongs to Uskov (1980) and Starykh (1986). The algorithms that determine the

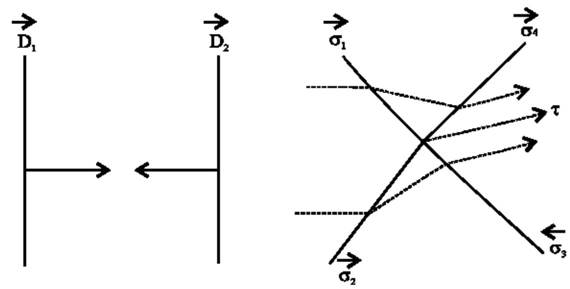

Fig. 1: Oncoming gas-dynamic discontinuities: a) one-dimensional shock waves and b) compression shocks: $\vec{D}_1$ "left" moving shock wave, $\vec{D}_2$ "right" moving shock wave, $\vec{\sigma}_1$ "left" incoming compression shock, $\vec{\sigma}_2$ "right" incoming compression shock, $\vec{\sigma}_3$ left" outcoming compression shock, $\vec{\sigma}_4$ "right" outcoming compression shock, τ-tangential discontinuity

type of interaction and nature of outcoming discontinuities have been developed by Adrianov (1988) and improved to a level of a package of applied programs for calculation of supersonic flows with shock waves in which all gas-dynamic discontinuities were defined and tracked clearly.

With new force, the interest in this problem arose at the end of 90's. It happened because of the start of hypersonic flight study programs. The first experiments were carried out with air intakes in which the interaction of oncoming compression shocks is realized has shown that change of flow velocity is provided with appearance of non-stationary and oscillation occurrences. With the increase of flow velocity this harmful to aircraft

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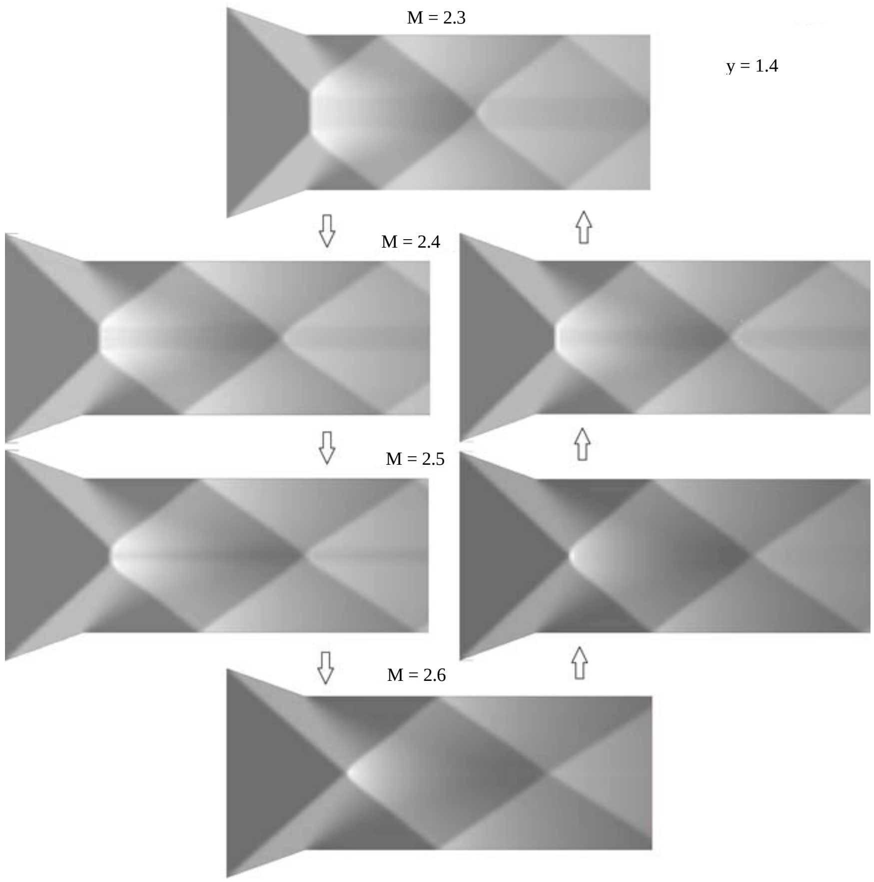

Fig. 2: Hysteresis during transition of regular oncoming shock interaction to irregular and back-wards

occurrences were more expressed. The analysis showed that the reason lies in solution ambiguity of gas-dynamic equations for regular and irregular shock interaction in some range of parameters which leads to hysteresis (Chpoun and Dor, 1995) at the increase or decrease of mach numbers before shock wave structure. This makes itself evident in the fact that during increase or decrease of mach numbers the transition from regular to irregular reflection occurs at different mach numbers (Fig. 2) and the mach stem differs as well. Advanced algorithms and methods were used to enhance calculation speed (Gaidhane and Hote, 2015; Sivasuthan *et al*., 2015).

The biggest investment into the study of this phenomenon was made by the researchers from the Institute of Theoretical and Applied Mechanics, Siberian Branch of the Russian Academy of Sciences (Ivanov *et al*., 2001; Dor 1991, 2001). It is worth to mention their co-work (Ivanov *et al*., 1999) in which the asymmetrical cases of oncoming shock interaction were studied. Dor *et al*. (2003) with use of numerical methods have studied the hysteresis during interaction of conical shocks.

Despite that the interaction of oncoming shock has being studied for >30 years, the theory is still unfinished. It is unclear if the classification of shock wave structures that occur during interaction of oncoming shocks, proposed by V.N. Uskov is complete. What are the criteria for transition from regular reflection to irregular? What it is the influence of dynamic effects and presence of disturbance before shock wave structure on hysteresis? All of this requires thorough analytical, numerical and experimental study.

The model of regular interaction of oncoming compression shocks: The interference type of oncoming compression shock $\vec{\sigma}_1$ and $\vec{\sigma}_2$ that have different direction, depends on their intensity J_1 and J_2, correspondingly. If J_1 and J_2 (Fig. 1b) is lower than J_s:

$$J_S = \frac{M^2-1}{2} + \sqrt{\left(\frac{M^2-1}{2}\right)^2 + \varepsilon\left(M^2-1\right)+1} \quad (1)$$

where, γ is specific heat ratio, then flow behind incoming shock is supersonic and as a result of their intersection the outcoming shocks appears $\vec{\sigma}_3$ and $\vec{\sigma}_4$, the direction of which are opposite to corresponding incoming shocks $\vec{\sigma}_1$ and $\vec{\sigma}_2$. Such interaction is called regular. The interference equation for supersonic oncoming shock in case of regular interaction looks like (Adrianov *et al*., 1995):

$$\beta_\sigma(\hat{M}_1, J_4) = -\beta_\sigma\left(\frac{J_1}{J_2}J_4, \hat{M}_2\right)+\beta_\sigma(M, J_1)+\beta_\sigma(M, J_2) \quad (2)$$

Flow deviation angles at crossed shocks is approximately equal to $\beta_3 \approx \beta_1$ and $\beta_4 \approx \beta_2$ at arbitrary intensity of incoming shocks. It is easy to demonstrate by using linear approximation for incoming shocks:

$$\Lambda = \ln J = \Gamma(M)\beta, \; \Gamma(M) = \frac{\gamma M^2}{\sqrt{M^2-1}} \quad (3)$$

From Eq. 3 follows:

$$\Lambda_3 = (a\Lambda_1+b)\Gamma_3(\hat{M}_1)/\Gamma_2(M) \quad (4)$$

Where:

$$a = \frac{\Gamma_2(M)+\Gamma_4(\hat{M}_1)}{\Gamma_4(\hat{M}_1)+\Gamma_3(\hat{M}_2)}; \; b = \frac{\Gamma_4(\hat{M}_1)+\Gamma_2(M)}{\Gamma_4(\hat{M}_1)+\Gamma_3(\hat{M}_2)}$$

By using linear dependencies between Λ_i and β_i:

$$\Lambda = \chi \times \Gamma(\gamma, M) \times \beta, \; \chi = \pm 1 \quad (5)$$

It is easy to find the turn angle on outcoming discontinuities:

$$\beta_3 = a\beta_1 - b\beta_2 \quad (6)$$

The coefficient b at low shock intensities is a value of second-order infinitesimal compared to coefficient α. This fact can serve as a basis for creating of quickly reconvergent algorithm for solving interference (Eq. 2). Assuming in zero approximation $\beta^0_3 = \beta_1$, we define intensity J^0_3 of reflected shock $\vec{\sigma}_3$ with use of cubic Eq. 7 with coefficients $A_n(\hat{M}_1, \beta_1)$:

$$\sum_{n=0}^{3} A_n z^n = 0 \quad (7)$$

Where:

$z = J-1$

$A_3 = 1+tg^2\beta$

$A_2(\beta; M) = (1+\varepsilon)\gamma M^2-[2\gamma M^2-(1+\varepsilon)]A_3$

$A_1(\beta; M) = \gamma M^2[\gamma M^2-2(1+\varepsilon)]tg^2\beta$

$A_0(\beta; M) = (1+\varepsilon)(\gamma M^2)^2 tg^2\beta$

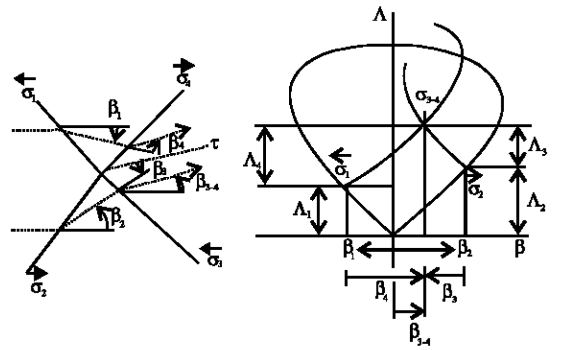

Fig. 3: The solution of problem of regular shock interaction on polar plane: left regular interaction of oblique compression shocks; right depiction of a regular interaction of oblique compression shocks on polar plane. β_i flow turn angle on shock number i, Λ_i intensity logarithm of shock number i, τ tangential discontinuity

Because:

$$J^0_4 = J^0_3 \frac{J_2}{J_1} \quad (8)$$

It is possible to find turn angle $\beta^0_4(\hat{M}_1, J^0_4)$ on shock $\vec{\sigma}_4$:

$$tg\beta_\sigma = \frac{1-E}{\sqrt{E}}\left(\sqrt{\frac{J_m-J}{1+\varepsilon J}}+\sqrt{\frac{1+\varepsilon J}{J_m-J}}\right)^{-1} \quad (9)$$

Index "m" is a maximal intensity of shock, i.e., intensity of a straight compression shock. If the difference $\Delta = (\beta_2+\beta^0_3)-(\beta_1+\beta^0_4)$ is higher than set calculation precision, then by setting in first approximation $\beta^{(1)}_3 = \beta^0_3+\Delta/2$ repeat the calculation procedure. Do note that at equal intensity of incoming compression shocks ($J_1 = J_2$) the intensities of outcoming shocks are equal to ($J_3 = J_4$) and their calculation is similar to the problem of shock reflection from a wall.

Dependencies $\Lambda = \ln J\ (\beta)$ at set Mach number are called shock polars or isomachs. The shock polars are convenient to use for analysis of discontinuity interference problems. Figure 3 shows graphical solution for cases of regular interaction of oncoming shocks. On the left of Fig. 3 the compression shocks σ_1 and σ_2 are showed that turn the original flow into opposite direction β_1 and β_2. The flows behind shocks σ_3 and σ_4 must be parallel to each other, thus the shock σ_4 turns flow in direction opposite to flow on shock σ_1. It is same with shock σ_2 and σ_3. The resulting turn angle $\beta_{3\text{-}4d}$ is determined as an algebraic sum of flow turn angles on all shock which is showed on Fig. 3 on the right. It is obvious that when the intensities of shocks σ_1 and σ_2 are equal, the resulting flow turn angle $\beta_{3\text{-}4}$ is equal to zero. The picture on polar plane is absolutely symmetrical and in this case, the interaction of oncoming shocks is similar to the case of regular shock reflection from a wall. The pressure behind shocks σ_3 and σ_4 must be equal, thus $\Lambda_1+\Lambda_4 = \Lambda_2+\Lambda_3$ which can be clearly seen on the right side of Fig. 3. It is obvious that at regular shock interaction the point 3-4 of transection of Polar 1 and 2 must be located inside the main polar. If secondary polar intersect above main polar then the regular and irregular shock interaction is possible.

The model of irregular interaction of oncoming shocks: If secondary polars that correspond to shock σ_1 and σ_2 do not intersect, then regular interaction between shocks is impossible. Obviously, the sufficient condition of solutions existence lies in location of limiting point (l) that corresponds to limiting slope angle of flow turn on one of the polar and inside another. These conditions can be formulated by comparing flow turn angles:

$$K_{l_1} = \beta_2(M, J_2)-\beta_3(\hat{M}_2, J_2J_{l_3})+\beta_1(M, J_1)-\beta_4(\hat{M}_1, J_2J_{l_3}) \quad (10)$$

Or:

$$K_{l_2} = \beta_2(M, J_2)-\beta_3(\hat{M}_2, J_1J_{l_4})+\beta_1(M, J_1)-\beta_4(\hat{M}_1, J_1J_{l_4}) \quad (11)$$

where, limiting angles $J_{l_3} = J_l(\hat{M}_2)$, a $J_{l_4} = J_l(\hat{M}_1)$ can be calculated with the help of Eq. 12:

$$J_l = \frac{M^2-2}{2}+\left[\left(\frac{M^2-2}{2}\right)^2+(1+2\varepsilon)(M^2-2)+3+2\varepsilon\right]^{1/2} \quad (12)$$

If $K_{l_1} \leq 0$ and the limitation $J_1J_{l_3} \leq J_5J_{m_4}$ is satisfied or $K_{l_2} \leq 0$ and the limitation $J_5J_{l_4} \leq J_1J_{m_3}$ is satisfied, then the solution to regular interaction exist. The maximum intensities of shock can be found using Eq. 13:

$$J_{m_4} = (1+\varepsilon)\hat{M}_1^2-\varepsilon,\ J_{m_3} = (1+\varepsilon)\hat{M}_2^2-\varepsilon \quad (13)$$

If Eq. 10-11 are not satisfied, then intersection point of polar 1 and 2 is located above main polar or polar 1 and 2 do not intersect at all. The boundary that separates Regular Interference (RI) and irregular Mach Interference (MI) is a case when intersection point of polar 1 and 2 is located on main polar (Fig. 4).

An analogue to this in case of shock reflection from a wall or symmetry axis is a Stationary Mach Configuration (SMC) when secondary polar intersects with main polar at its apex (Uskov *et al*., 2012a, b). If the intensities of incoming shocks are equal to $J_1 = J_2$, then this point is located at the apex of main polar and the transition RI→MI starts at $J_1 = J_2 = J_0$, J_0 and is defined by Eq. 14:

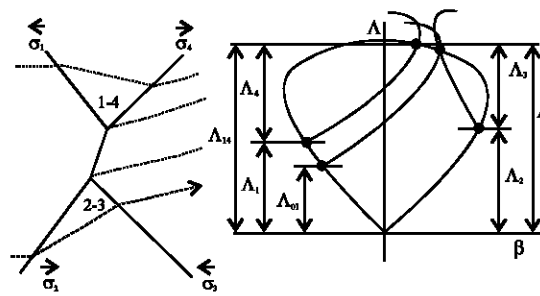

Fig. 4: Solution of problem of mach interaction of shocks on polar plane: left irregular interaction of oncoming shocks; right depiction of solution of oblique oncoming shocks on polar plane; Λ_i intensity logarithm of shock number i. Λ_{01} logarithms of special shock intensity $\bar{\sigma}_1$ at the transition from regular to irregular interaction occurs

$$\sum_{k=0}^{3} A_kJ_0^k = 0,\quad \begin{aligned} A_3 &= 1-\varepsilon^2 \\ A_2 &= -\left(\left(1+\varepsilon-\varepsilon^2+\varepsilon^3\right)J_m+1+\varepsilon^2\right) \\ A_1 &= \varepsilon(1+J_m)\left[(1-\varepsilon)J_m-2\right] \\ A_0 &= (1-\varepsilon)J_m(J_m-1) \\ J_m &= (1+\varepsilon)M^2-\varepsilon \end{aligned} \quad (14)$$

In case $J_1 \neq J_2$ for each value of J_2 there is a corresponding value of J_{01} that defines a start of irregular interference (Fig. 4). This value can be defined by a following procedure: using preset value of M and J_2 a triple shock wave configuration is calculated (TC-1 or TK-2 by classification (Uskov and Chernyshov, 2006) and the intensity $J_{1\text{-}4}$ of main shock is found in this configuration. The value $J_{1\text{-}4}$ by using method of calculating triple configuration with shock $\bar{\sigma}_1$ allows to find intensity J_{01} of other compression shock at which the given triple configuration exist. It is assumed that main shock is straight $J_{1\text{-}4} = J_{2\text{-}3}$. This is how the existence region of irregular interaction of oncoming compression shock is defined. In the case described above the transition RI→MI occur fluently without sudden changes of parameters.

It is worth noting that there is a range of problem parameters when secondary polar intersect above main polar. For a long time it was assumed that in this case, one should choose MI. However, the late numerical and experimental results appeared showing that realization of either solution depends on pretext, i.e., the direction of problem parameters change and presence of disturbances. Thus, theoretically both solutions can exist in the given region: RI and MI. Its boundary is a case when polars touch and their touch point is located outside of main polar. The criterion and mechanism of reverse transition MI→RI in a region of solution uncertainty requires addition research.

A touch of polars at points that correspond to limiting flow turn angles on corresponding shock can occur inside of main polar. For this to occur its sufficient that limiting point of either of polar that corresponds to $J = J_1$ was located inside of main polar. If a contact point of both polars located inside main polar that fluent transition to MI is impossible, i.e., during degradation of intersection point of secondary polar the shock wave structure suddenly transfer to a configuration that corresponds to intersection of secondary polars with subsonic part of main polar.

RESULTS AND DISCUSSION

Definition of outcoming shock intensity during RI: If during RI we're to read the intensity of a shock for instance σ_1 and increase the intensity of shock σ_2, the intensity of crossed shocks would also increase (in a given case σ_3) and the intensity of an adjacent shock σ_4 would decrease. The dependence of intensities of shocks σ_3 and σ_4 on intensity of interaction shocks σ_1 and σ_2 is presented on Fig. 5. As we can, the intensity of crossed compression shocks can differ quite significantly.

Definition of limiting parameter during transitions RI→MI: Some result of calculating dependency $J_{02}(J_1)$ for γ = 1.4 and various mach number presented on Fig. 6. Straight line corresponds to equation $J_1 = J_2$.

A change of quality picture of flow at the increase of intensity of one of the incoming shocks: Let us look at transition of regular oncoming compression shock interference into a mach interference throughout increase of incoming shock wave interference (Fig. 7).

At high mach numbers and fixed intensity of one of the shocks J_1 with the increase of value J_2 the quad shock regular configuration of shack waves (Fig. 7a) transforms into quint shock mach configuration (Fig. 7b) at $J_2 = J_0$ with the appearance of bridge-like shock wave σ_5 that generates triple configurations TC-2 at the point 2-3 at shock σ_2 and TC-1 at the point 1-4 at σ_1. While its form is straight and intensity is $J_5 = J_{1\text{-}4} = J_{2\text{-}3}$. Further increase of J_2 leads to the transformation of TC-2 at constant

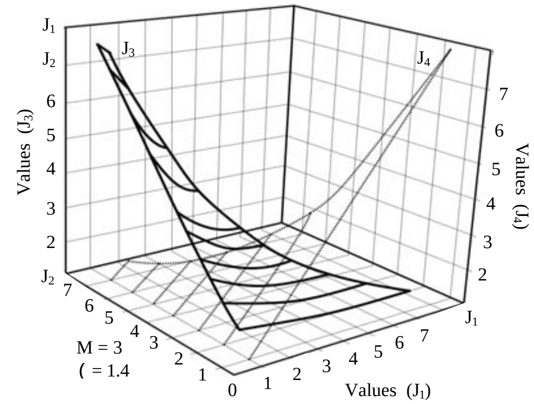

Fig. 5: Dependence of reflected shock intensities on intensity of incoming shock at regular interaction

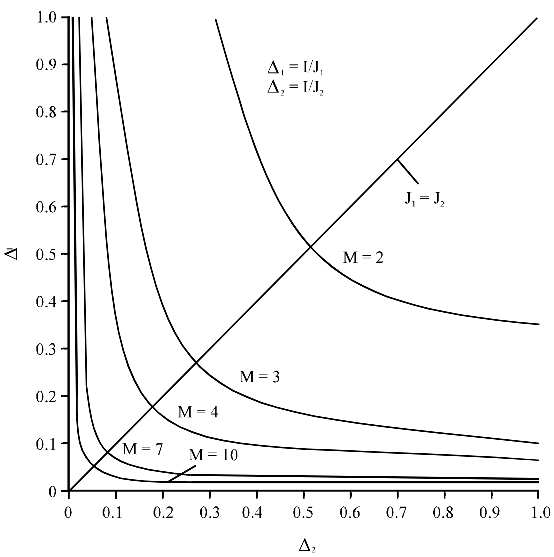

Fig. 6: Dependency of incoming compression shock intensity that corresponds to a start of MR on intensity of other incoming shock

intensities of shock waves, into TK-1 because of that the shock σ_5 becomes curved (Fig. 7c). Its length increases. The intensity of shock σ_5 changes from $J_{1\text{-}4}$ to $J_{2\text{-}3}$. If we were to increase intensity of shock σ_2 to $J_2 = J_T$ where:

$$(1+\varepsilon)M^2(1+\varepsilon J_T)^2 = (1-\varepsilon)(J_T+\varepsilon)\times \left[(1+\varepsilon)M^2-(J_T+\varepsilon)\right]\left[(J_T+\varepsilon)M^2-(J_T-1)(J_T+2-\varepsilon)\right] \quad (15)$$

When $J_2 = J_T$, the reflected compression shock σ_3 becomes straight and changes its direction generating TC-3, i.e., shocks σ_2 and σ_3 form a structure that consist of overtaking shock of same direction.

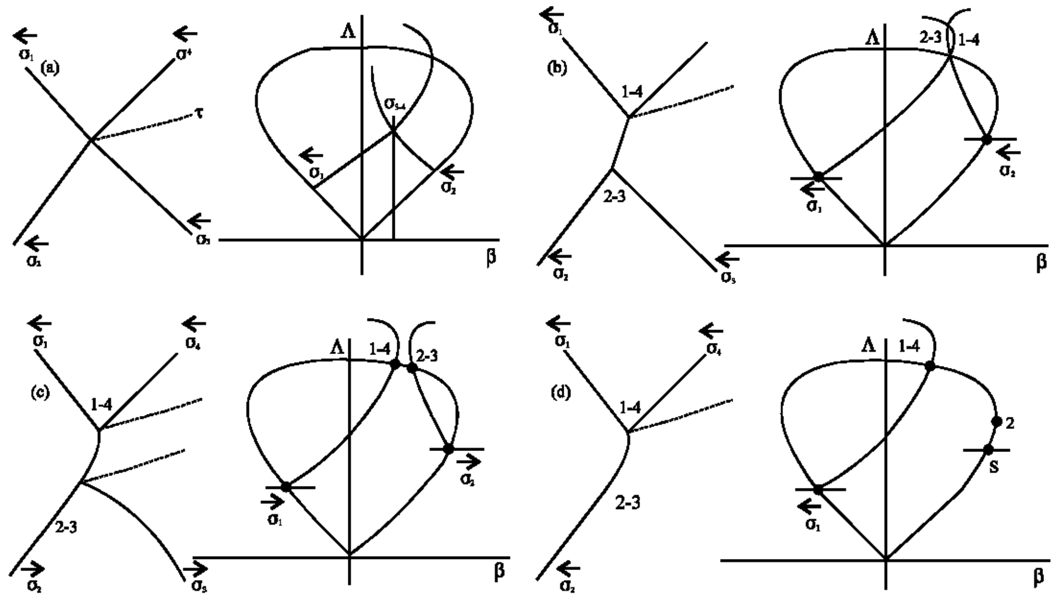

Fig. 7: Transformation of shock wave structure with a change of intensity of one of the two interacting shocks: a) Regular interaction; b) Mach reflection with straight main compression shock; c) Mach reflection with bridge-like main compression shock and d) Configuration with a degenerate triple point 2-3

If we were to further increase the intensity of shock σ_2 a way that flow behind it becomes subsonic at $J_2>J_S$, shock σ_3 disappears and interference of oncoming shock is expressed in a form of singular triple configuration TC-1. The incoming compression shock σ_2 becomes curved and its intensity changes from J_2 to a value that is equal to the intensity of shock 1-4 in TC-1. If we were now to increase the intensity J_1 of second incoming compression shock σ_1, then at $J_1>l_S$, then the triple configuration at this shock disappears and interference of oncoming shock wave leads to a generation of singular curved compression shock with fully subsonic flow behind it.

The rearrangement of shock wave structure of oncoming shock at low mach number wasn't reviewed here. At symmetrical interaction of oncoming shock, the shock wave structures are similar to ones depicted on Fig. 7 but symmetrical.

CONCLUSION

The interaction of oncoming shock in cases that are more complex than compression shock reflection from a wall was reviewed. At regular interaction of oncoming shock, the intensities of crossed shocks can vary quite significantly. Transition from regular to irregular reflection can occur smoothly at high mach number or in a form of a sudden non-stationary process at low mach numbers. The mechanism of irregular interaction at low mach number requires a more detailed study. The transformation of shock wave structure with increase or decrease of mach number is characterized by presence of hysteresis, i.e., in some range of parameters; two different pictures of flow can correspond to a certain parameter of problem. A range of problem parameters in which the existence of regular and irregular reflection of shock is assumed, explains a possibility of hysteresis appearance.

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