Refinery Scheduling for Naphta Hydrotreater Unit using Non-Linear Programming

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Abstract: In a refinery industry, production scheduling is one of the most critical activities in entire process. Expensive raw material prices, high operational costs and limited amount of resources enforce this industry to make their production schedule properly. In scheduling, the quantity of the product is not the only matter. We should also consider the quality of the resulting product that must be in accordance with the specified standards. The object of study in this study is Naphtha Hydro Treater (NHT) with the naphtha input. This study considers a refinery scheduling using a non-linear programming model. The channel structure of production system consists of three echelon parties, i.e., vessel, storage tanks and NHT unit.

Key words: Scheduling, naphtha, non-linear programming, model, channel, NHT

INTRODUCTION

Crude oil scheduling is an important decision of the refinery supply chain (Saharidis and Ierapetritou, 2009a). Crude oil scheduling is a process that involves specifying the timing and sequence of operations in the order of vessel arrival, crude oil unloading to storage tanks, transferring crude oil from storage tanks to charging tanks and finally sent the mixed crude oil to Crude Distillation Units (CDUs) for component separations and downstream processing. A typical schedule sets daily targets for production with consideration on storage and changing tanks capacities, crude distillation capacity utilization and pumping capabilities (Hamisu *et al.*, 2013). It also determines the quality and quantity of crude mixing materials in the changing thanks in order to produce blends that satisfying the product requirement.

Before discussing about the scheduling of refinery production there are several things that are necessary to be explained first. In this study, production planning will be performed on refinery industry, specifically on the processing unit called Naphtha Hydro Treater (NHT). This unit is used or assigned to convert naphtha into light naphtha and heavy naphtha and then processed back in the next unit and converted into High Octane Mogas Component (HOMC). Later, HOMC will be used as the main raw material manufacture of Premium gasoline or Pertamax manifold, one of lubricants products.

Naphtha is obtained in petroleum refineries as one of the intermediate products from the distillation of crude oil. It is a liquid intermediate between the light gases in the crude oil and the heavier liquid kerosene. Naphtha is a volatile, flammable and has a specific gravity of about 0.7. The generic name naphtha describes a range of different refinery intermediate products used in different applications. To further complicate the matter, similar naphtha types are often referred to by different names (Hidayat *et al.*, 2013). The different naphtha is distinguished by density (g/mL or Specific Gravity (SG)), PONA (Paraffins, Olefins, Naphthenes and Aromatics), PIONA (Paraffins, Isoparaffins, Olefins, Naphthenes and Aromatics) or PIANO (Paraffins, Isoparaffins, Aromatics, Napthenes and Olefins) analysis.

In this study, we concern with Specific Gravity (SG) aspect mentioned above. It is due to the reason that number of SG affects a number of octanes in the end of the product. Subsequently, PONA is considered as a given aspect because in a short-term it has less impact on the product.

The production scheduling system under discussion includes loading and unloading mechanisms in the unit. The loading process is the process of moving the contents of the vessel to the three tanks whereas the unloading process is the process of withdrawal of naphtha from tank to NHT unit for blending processing.

The raw material of each unit can be distributed using a variety of facilities such as pipeline, truck or vessel. Naphtha which is the raw material of NHT unit is transported by vessel. Submitted naphtha has a different charge and value of SG for every vessel, depending on

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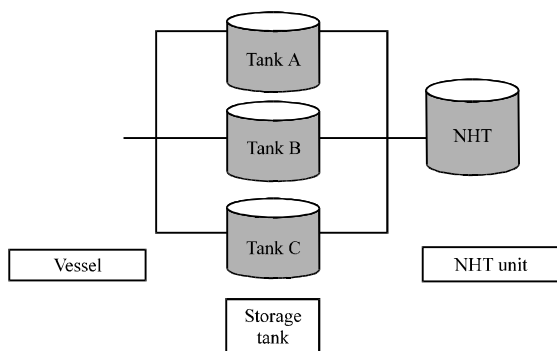
Fig. 1: A scheme of Naphtha Hydrotreater Unit (NHT)

the origin of the naphtha sources. Naphtha cargo from each vessel ranges between 70-200 MB and value of SG range between 0.68-0.74.

The distribution scheme of NHT involves a three-echelon distribution system. According to Eq. 3 from the operational side, cost, flexibility, quality and delivery take precedence in prioritizing importance. Just In Time (JIT) is used to increase company performance. The application of Value Chain Management (VCM) is used in every company in a supply chain network in order to develop an efficient method to reduce inventory cost to increase response towards demand change and to increase consumer's satisfaction. The distribution scheme of NHT is shown in Fig. 1.

Naphtha which is transported by a vessel will be accommodated prior to the three tanks (Fig. 1). All three tanks have a maximum capacity of 270 MB and a limit of 47 MB un-pump-able naphtha. Un-pump-able limit is the rest of naphtha that cannot be pumped out of the tank due to the facts that not all tanks can accommodate naphtha cargo carried by the vessel. In brief, tank used as a source of feed for NHT unit (or is said to be in operation) cannot accommodate naphtha simultaneously. Or in other words, the tank that is not in operation (empty or idle) can accommodate naphtha. In our case study, there are several numbers of tanks that operate every day.

Furthermore, after naphtha is stored in the tanks, the naphtha will be used as a source of feed at NHT unit. Within a day, NHT unit can operate and process 42 MB of naphtha. The processing takes place every day. Therefore, in every day, it should be 42 MB of naphtha flowed from the tank to the NHT unit. The naphtha can be sourced from a single tank or two tanks, depending on the number of naphtha which is still available on the tank. When the number of naphtha which can be drawn from the feed source is <42 MB then there will be the next tank is the source of feed.

Rule of thumb in selecting the tank is seen from the feed source and SG naphtha tank capacity in the tank. Tank that contains the largest amount of naphtha will be prioritized to be used as a source as well as the certain minimum number of SG on naphtha is 0.71. The number of SG on naphtha suggested by the Indonesia government is 0.72.

In the production scheduling we got an initial input which is a form of scheduled arrival of a vessel called the offtake data. This offtake data includes the date of arrival of the vessel for the next month that is completed with a large payload and content of SG. The goal of this production scheduling is to determine whether this allocated resource plan can be executed at the refinery or not. The way to ensure is by doing the optimization of production of naphtha processing to determine the optimal allocation of naphtha in each tank every day.

This proposed model is solved by using an Add-ins on Microsoft Excel, solver. The advantage of using solver is the ease of visualization and modeling itself. While strapped of using solver is a limited ability to search the optimal solution.

Literature review: This problem of scheduling of production in petroleum has appeared for long time ago, since the introduction of linear programming (Floudas, 1995b). Many of them make a crude distillation unit as an object for an example. We distinguish this study to other studies by exploring the different object and the tools to solve the problem. Saharidis *et al.* (2009b) addressed that the developed methods for the resolution of this problem are classified into three general groups, i.e., the exact methods that use either discrete or continuous time representation and the heuristic methods. This study uses an exact method that uses discrete time.

According to Shah (1996) in the system under discussions, the available tanks can feed only one crude distillation unit at a time and a crude distillation unit can be fed by only one tank at a time. In order to accommodate this situation, they subdivide the scheduling time horizon into intervals of equal duration where each activity must start and finish within the boundaries of these intervals. On the other hand we propose algorithms that make it possible to accommodate several tanks simultaneously.

Related to the constraints, some linearity assumptions are maintain by Lee *et al.* (1996) by replacing the bi-linear terms due to several tanks mixing operations in to the individual component flows. Lee and Kim (2002), all the non-linear constraints associated with the blending operations were linearized. According to Reddy *et al.* (2004), these types of linearization raise the blend failure and unsatisfied crude distillation unit operational constraints.

In finding the solution, sequential procedures were developed by Joly *et al.* (2004) assigned in two models, the first for planning and the second for scheduling in refinery. In the refinery scheduling a linear model is utilized for the loading and unloading of crude oil between the tankers and the crude distillation unit.

Hadi *et al.* (2013) developed a scheduling model for job shop production system with total actual flow time as the performance criteria considering earliness and tardiness problems on production activity. During production process, the operator uses basket as tools of material handling. The basket is utilized to store and to move work-in-process and finished goods and it has limited capacity. This material handling process becomes important. This is due to the fact that in the common manufacturing strategy characteristics when the order has a high probability of arriving before the procurement (material supply) process, the best manufacturing strategy is Make To-Order (MTO) and this condition also strengthens the procurement probability of components (Hidayat *et al.*, 2011). We adapt the concepts of MTO production system and earliness-tardiness concepts into our system under discussion.

In this study, it is considered a Mixed Integer Non-Linear Programming (MINLP) Model because there are some constraints that are non-linear and some variables that should be integer in a simultaneous procedures. By definition, non-linear programming is a solving optimization problem defined by a system of equalities and inequalities, collectively termed constraints, over a set of unknown real variables along with an objective function to be maximized or minimized where some constraints or objective functions are non-linear (Bertsekas, 1999). Wenkai *et al.* (2002) stated that when using a non-linear problem there will be no guarantee in finding feasible solutions.

MATERIALS AND METHODS

Mathematical model: As explained before, our planning for production schedules will be solved using a non-linear programming optimization. In the system there is some useful information that can help to develop an optimization model such as: limits on the production process (maximum capacity, maximum production capacity, flow rate of unloading naphtha). Offtake data (scheduled arrival of the vessel, the amount of naphtha transported, origin arrival of the ship, SG naphtha transported). The last condition system at the end of the

Table 1: List of mathematical notations

Index	Description
t	Time for loading and unloading $t = \{1, 2, ..., n\}$
T	Naptha storage tank $T = \{A, B, C\}$
N	Number of time
Parameter	**Description**
Vessel load (t)	Total naphtha that allocated from vessel at time t (MB)
SG from vessel (t)	SG that contain in naphtha from vessel
Feed per day (t)	Total feed of naphtha to NHT for every t (constantly 420 MB)
Initial stock (1, T) (MB)	The amount of naphtha on the tank T at time $t = 1$ (MB)
Initial SG (1, T)	Initial SG to tank T at time $t = 1$ (MB)
Status (1, T)	status for tank T at $t = 1$ (integer)
Max capacity	The maximum volume of the tank T at time t (MB)
Min capacity	The minimum volume of the tank T at time t (MB)
Min SG combine	Minimum SG number that allowed for entry in to NHT unit at time t
SG Min	Min SG for each tank
Decision variable	
Allocated naphtha time t (t, T)	Amount of naphtha that allocated in tank T at (MB)
Variable	
Initial stock (t, T)	Initial stock for tank T at time t (MB)
Initial SG (t, T)	Initial SG for tank T at time t
Status (t, T)	Status for tank T (0 = stand by; 1 = feed) (integer)
Ullage (t)	Ullage or an empty space in both tanks at time t (MB)
Discharge (t, T)	Amount of discharge that allocated to tank T at time t (MB)
End SG (t, T)	Last value of SG for tank T at the end of time t
End stock (t, T)	Remaining stock in tank T at the end of time t (MB)
Status of source feed (t, T)	Status that shows the source of feed (0 = not a source feed; 1 = potential source feed; 2 = feed sources are nearly run out) (int)
Feed source (t, T)	shows the amount of naphtha is drawn on the tank T to feed in to NHT at time t (MB)
SG combine (t)	shows SG number of naphtha that goes into NHT unit (from tank/tanks that status is 'feed') at time t
Shortage (t)	Amount of shortage of naphtha at time t (MB)
Total feed (t)	Total amount of naphtha that entry in to NHT unit (MB)

month, the end of the production period (SG naphtha in each tank, the volume of naphtha in each tank, tank status feed).

As said before the most affecting aspect for the quality of production is the value of SG on naphtha. The smaller number of the SG on naphtha leads to the lesser octane value generated in the end of the product. But the number of SG that are too high will provide low opportunity costs for the company. So, the company considers optimal SG on naphtha range between 0.71-0.73 with the ideal number is selected as 0.72. Departing from it all, the optimization model is made in the form of minimizing the objective function gap among number of SG that goes into NHT unit with ideal SG which is 0.72.

Mathematical notations: In developing the mathematical model to representing the problems we use notations consisting indexes, parameters, variables and decision variables as shown in Table 1.

Table 2: Characteristics of naphtha for each vessel

Vessel	Origin	Cargo (MB)	SG
1	RU III	70	0.74
2	RU IV	100	0.68
3	RU V	200	0.72

- Unit NHT always produces at maximum capacity every day which is equivalent with feed per day (Table 2)
- Selection of the tank as a feed based on the rule of thumb, i.e., selecting naphtha tank that has the most numerous and SG are eligible
- Model allows source feeds from more than one tank

Equations: The objective function of this problem is as follows:

$$\text{Min } Z = \left[\frac{\sum_{t=1}^{n} \text{SG Combine}(t)}{n} - 0.72 \right] \quad (1)$$

These following constraints are developed to: ensure the final volume of the tank does not exceed the maximum limit of the tank:

$$\text{End stock}(t, T) \leq \text{Max capacity}\left(t = \{1, ..., n\}; T = \{A, B, C\}\right) \quad (2)$$

Ensure the final volume of the tank is not less than the minimum tank:

$$\text{End stock}(t, T) \geq \text{Min capacity}\left(t = \{1, .., n\}; T = \{A, B, C\}\right) \quad (3)$$

Ensure that the amount equals to the total allocation discharge at the time t:

$$\sum_{t=A}^{C} \text{Discharge}(t, T) = \text{Vessel load}(t)\left(t = \{1, ..., n\}\right) \quad (4)$$

Ensure SG goes into NHT greater than or equals to SG combine min for each t:

$$\text{SG combine}(t) \geq \text{Min SG combine}\left(t = \{1, .., n\}\right) \quad (5)$$

Ensure SG in the tank T greater than SG min for every tank T and every time t:

$$\text{SG End}(t, T) \geq \text{SG min}\left(t = \{1, .., n\}; T = \{A, B, C\}\right) \quad (6)$$

Restrictions on the amount of feed to the NHT to match the amount in the feed every day:

$$\sum_{t=A}^{C} \text{Feed source}(t, T) = \text{Feed per day}(t)\left(t = \{1, ..., n\}\right) \quad (7)$$

Algorithm: To obtain an optimal solution then do the following algorithm from Step 1 until 18:

Input schedule of vessel arrival for a month ahead that constraints the date of arrival, vessel load (t) and SG from vessel (t).
Determine the initial stock for each tank using Eq. 8:

$$\text{Initial stock}(t, T) = \begin{cases} t = 1; \text{input parameter of initial stock}(1, T) \\ t > 1; \text{end stock} \end{cases} \quad (8)$$

for each t and T

Initial stock is a volume of naphtha for each tank at the beginning of time t.
Calculate the Ullage using Eq. 9:

$$\text{Ullage}(t) = \sum_{T=A}^{C} (\text{Max capacity} - \text{Initial stock}(t, T)) \quad (9)$$

for each t

Ullage is an empty space on the whole tank, equivalent to the amount of naphtha which can be accommodated in the tank.
Calculate the initial SG for each tank using Eq. 10:

$$\text{Initial SG}(t, T) = \begin{cases} t = 1; \text{input parameter of initial SG}(1, T) \\ t > 1; \text{End SG}(t-1, T) \end{cases} \quad (10)$$

for each t and T

Initial SG is a SG number of naphtha on each tank at the beginning of time t.
Determine the status for each tank at time t using Eq. 11:

$$\text{Status of feed sources}(t, T) = \begin{cases} \text{Status} = 1; \begin{cases} \text{End stock}(t, T) - \\ \text{Feed per day} < \text{Min capacity}; 2 \\ \text{End stock}(t, T) - \\ \text{Feed per day} > \text{Min capacity}; 1 \end{cases} \\ \text{Status} = 0, 0 \end{cases}$$

for each t and T

$$(11)$$

The status indicates the operating status of each tank at time t:
Calculate the discharge using Eq. 12:

$$\text{Discharge}(t, T) =$$
$$\begin{cases} \text{Status} = 1; 0 \\ \text{Status} = 0; 0 < \text{Discharge}(t, T) < \text{Allocated naphta}(t, T) \end{cases} \quad (12)$$

for each t and T

Discharge is an allocation amount of naphtha that goes into the tank T at time t.
There are several parameters that we use in this model such as feed per day which is amount of naphtha that regularly comes in to NHT. There are several other parameters such as capacity Min, capacity Max, SG Min and SG combine Min (Table 2).

Calculate the end SG using Eq. 13:

$$\text{End SG (t, T)} = \frac{\left[\begin{array}{l}\left(\text{Initial SG (t, T)} \times \text{Initial stock (t, T)}\right) + \\ \left(\text{SG from vessel (t)} \times \text{Disch arg e (t, T)}\right)\end{array}\right]}{\text{Initial stock(t, T)} + \text{Disch arg e(t, T)}} \quad (13)$$

for each t and T

End SG is a SG number of naphtha on each tank at the end of the day. Calculate the end stock using Eq. 14:

$$\text{End stock (t, T)} = \begin{cases} \text{Status} = 1; \text{Initial stock (t, T)} - \text{Feed per day} \\ \text{Status} = 0; \text{Initial stock (t, T)} + \text{Disch arg e(t, T)} \end{cases} \quad (14)$$

for each t and T

End stock is a volume of naphtha for each tank at the end of time t. Determine the status of source feed using Eq. 15:

Status of feed sources (t, T) =

$$\text{Status} = 1; \begin{cases} \text{End stock (t, T)} - \text{Feed per day} < \text{Min capacity; 2} \\ \text{End stock (t, T)} - \text{Feed per day} > \text{Min capacity; 1} \end{cases} \quad (15)$$

Status = 0, 0   For each t and T

Status of source feed is an indicator status to determine whether the current feed would have shortage of naphtha or not. (Index of status: (0) standby (Eq. 1) feeds (Eq. 2) there is a shortage of naphtha to feed)

Determine amount of Shortage using Eq. 16:

$$\text{Shortage (t)} = \begin{cases} \text{Status of feed sources} = 2; \text{End stock(t, T)} - \text{Min capacity} \\ \text{Status of feed sources} < 2; 0 \end{cases}$$

for each t

$$(16)$$

Shortage is an amount of shortage of naphtha which cannot be met by the current tank.

Determine the feed source using Eq. 17:

Feed sources (t, T) =

$$\begin{cases} t = 1 \begin{cases} \text{Status} = 1; \begin{cases} \text{Initial stock (t, T)} - \text{Feed per day} \leq \text{Min capacity;} \\ \text{Initial stock (t, T)} - \\ \text{Min capacity initial stock (t, T)} - \text{Feed per day} > \\ \text{Min capacity; Feed per day} \end{cases} \\ \text{Status} = 0; 0 \end{cases} \\ t > 1 \begin{cases} \sum_{T = A}^{C} \text{Status(t, T)} = 1; \begin{cases} \text{Status (t} - 1, T) = 1; \text{Feed per day} \\ \text{Status (t} - 1, T = 0; \text{Feed per day} \\ -\text{Shortage(t)} \\ \text{Status (t, T)} = 0; 0 \end{cases} \\ \sum_{T = A}^{C} \text{Status(t, T)} = 1; \begin{cases} \text{Status (t, T)} = 1; \text{Feed per day} \\ \text{Status (t, T)} = 0; 0 \text{ for each t} \end{cases} \end{cases} \end{cases}$$

$$(17)$$

Sources feed indicate the number of naphtha which feeds into the NHT from of each tank at time t.

Calculate total feed at time t using Eq. 18:

$$\text{Total feed (t)} = \sum_{T = A}^{C} \text{Feed sources (t, T)} \quad (18)$$

for each t

Total feed is an amount of naphtha that feed into NHT at time t.

Table 3: Initial condition of tank

Tank	Max capacity (MB)	Initial volume	Initial status	Initial SG
A	270	251	Feed	0.742
B	270	84	Stand by	0.724
C	270	257	Stand by	0.718

Calculate the SG combine using Eq. 19:

$$\text{SG combine (t)} = \frac{\sum_{T = A}^{C} \{\text{Status (t, T)} \times \text{Feed sources(t, T)} \times \text{End SG (t, T)}\}}{\sum_{T = A}^{C} \text{Feed sources(t, T)}}$$

for each t

$$(19)$$

SG combine shows the value that goes to the NHT unit at time t.

Determine objective function using (Eq. 1). This objective function will be used to minimize the difference between the average of SG number that entered into NHT and SG ideal.

The decision variable for this model is allocated naphtha (Table 3). A decision variable in this model is the optimal number of naphtha allocation on a particular tank.

Constraints that are used in this model in Eq. 2-7.

To find an optimal solution, it is used solver by Microsoft Excel as tools with the solving method is GRG non-linear.

RESULTS AND DISCUSSION

A numerical example: A case study that used for this study is a real refinery scheduling that has been done for last July 2014. Naphtha that be processed comes from three different regions that are represented by three different vessels (Table 4). However, in our calculation we only need the characteristics of naphtha such as large cargo and SG number of naphtha. Business process and related equations are shown in Fig. 2.

In order to test the proposed model and algorithm in Table 2, we will get the initial conditions for each tank such as the initial volume of each tank, the status of each tank and the initial SG number of naphtha. There is also a scheduled arrival of vessels as shown in Table 3. Optimization using solver from Microsoft Excel, the results are shown in Fig. 3 and 4.

From the result above, there are one arrival of vessel that cannot be processed due to ullage or empty space of the tank is not sufficient to accommodate the naphtha. This vessel was supposed to bring as much as 200 MB naphtha on July 9-10th. However, according to ullage on that day with tolerance around 2 days, it still cannot accommodate 200 M naphtha. So, we suggest making a switch vessel with fewer load of naphtha, e.g., 100 MB. In the calculation above we consider the arrival of vessel loads 100 MB and has obtained a sub-optimal solution. Therefore, when calculated, total amount of naphtha that can be processed by calculation above is about 1.290 MB in which the plan allocation is 1.390 MB.
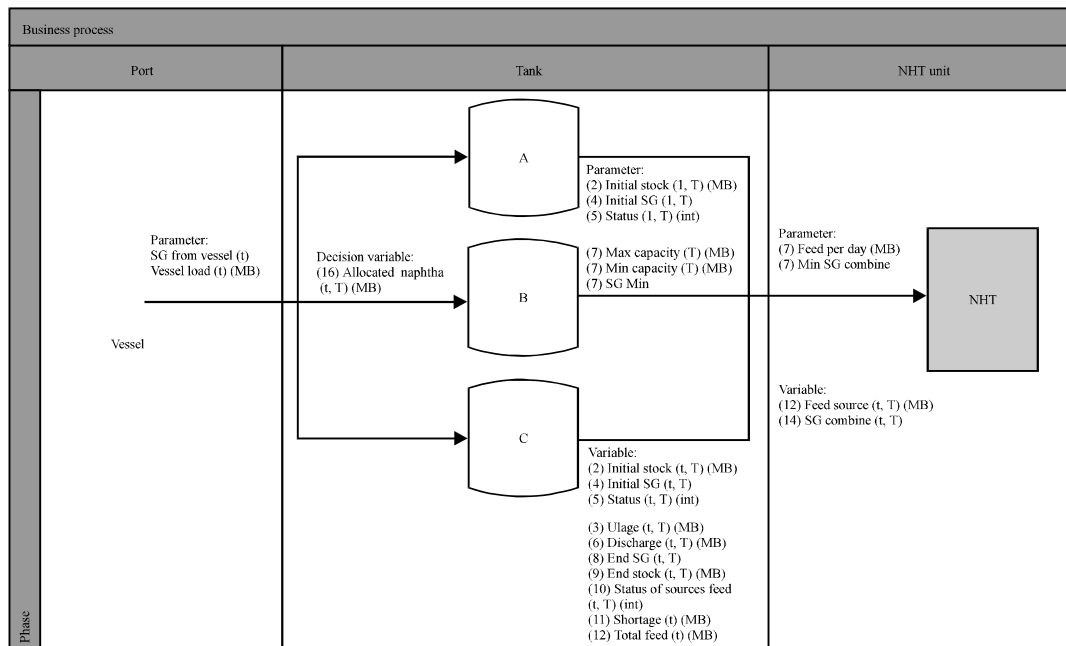
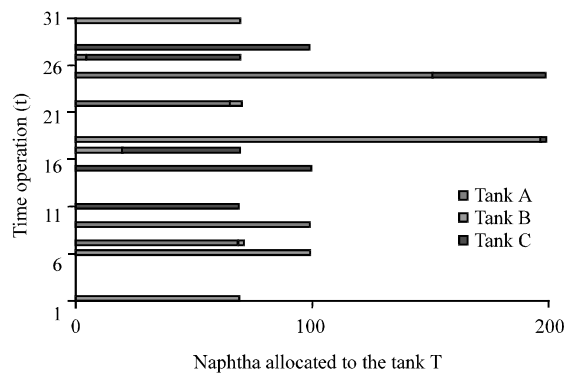Fig. 2: Business process and related equations

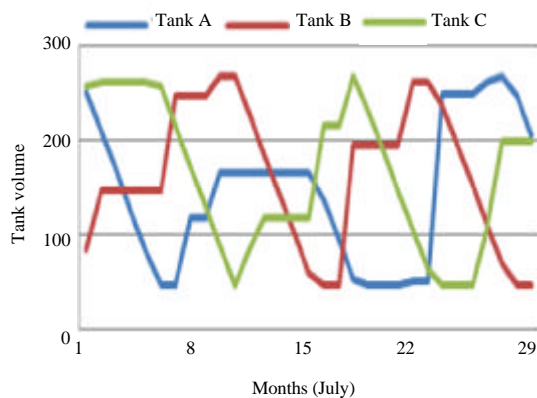Fig. 3: Optimal allocation of naphtha for every tank for July, 1st to 31th

Fig. 4: Graphic of volume for each tank

Table 3: Initial condition of tank

Tank	Max capacity (MB)	Initial volume	Initial status	Initial SG
A	270	251	Feed	0.742
B	270	84	Stand by	0.724
C	270	257	Stand by	0.718

Table 4: Time of arrival of vessels

Time of arrival (t)

Vessels	Vessel 2	Vessel 3
1-2	6-7	8-9
7-8	15-16	18-19
11-12	26-27	24-25
17-18		
22-23		
27-28		
31		

CONCLUSION

The study considered a refinery scheduling of Naphtha Hydro Treater (NHT) using a non-linear programming model for NHT with the Specific Gravity (SG) aspect. Several linear constraints were considered to avoid the blend failure and the unsatisfied crude distillation unit operational constraints. In finding the solution, the algorithm used simultaneous procedures assigned in two models, the first for planning and the second for scheduling in refinery. In the refinery scheduling a non-linear model for the loading and unloading of crude oil between the tankers and the crude distillation unit was considered. The simulation results showed the optimum solution for the proposed model where only one arrival of vessel that cannot be processed due to the ullage or the empty space of the tank was not

sufficient to accommodate the naphtha due to the quality of the resulting product that must be in accordance with the specified standards.

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